Newton and the Greeks

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Isaac Newton is Isaac Newton. His physics, mathematics, and language have profoundly shaped the modern world. So much so, in fact, that his once radical ideas now seem natural and inseparable from our own. This makes it hard to discern which of his intuitions were once counterintuitive or otherwise special.

This leads us to wonder: What, exactly, makes Newton's intuition so exceptional?

In this paper, we will explore the origins of Newton's remarkable intuition in physics, tracing how Greek geometry, Aristotelian philosophy, and the Atomist tradition set the stage for Newton's achievements. First, we briefly state Einstein's praise for Newton's "differential law" as a way of contextualizing our discussion. Next, we discuss Newton's geometry beginning with Feynman's perspective. We then compare Aristotle's final-cause-driven worldview with the mechanical worldview associated with ancient Atomists and Newton. Finally, we argue that Newton's true genius lay in his "clear conception of the differential law" as stated by Einstein, shaped by a sustained, methodical approach rather than a single stroke of insight.

> It is just two hundred years ago that Newton closed his eyes. It behooves us at such a moment to remember this brilliant genius, who determined the course of western thought, research and practice to an extent that nobody before or since his time can touch.
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> No doubt the great materialists of ancient Greece had insisted that all material events should be traced back to a strictly regular series of atomic movements, without admitting any living creature's will as an independent cause. And no doubt Descartes had in his own way taken up this quest again. But it remained a bold ambition, the problematical ideal of a school of philosophers. Actual results of a kind to support the belief in the existence of a complete chain of physical causation hardly existed before Newton.
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> The differential law is the only form which completely satisfies the modern physicist's demand for causality. The clear conception of the differential law is one of Newton's greatest intellectual achievements. It was not merely this conception that was needed but also a mathematical formalism, which existed in a rudimentary form but needed to acquire a systematic form. Newton found this also in the differential and the integral calculus. (Einstein, 2009, 28-30)

Einstein distinguishes Newton's achievement of a mathematical formalism of the ‘differential law’ in the *Principia* from the mathematics of Newton’s differential and integral calculus. We will take this distinction up later in the paper. Nowadays, though—for good reason we might add—this distinction is less emphasized, and we more freely associate Newton’s Laws of Motion and Universal Law of Gravitation with their expressions in terms of differential and integral calculus. Using calculus has the advantage of making it easier to analytically calculate the effect of all forces on motion.

Modern scientists echo this sense of awe at Newton’s insight. But, modern viewpoints risk oversimplifying Newton’s thinking.

Steven Strogatz, who studied at Trinity College—Newton’s own college—emphasizes how calculus and Newton’s laws of motion unify seemingly disparate phenomena:

> Isaac Newton was the first to glimpse this secret of the universe. He found that the orbits of the planets, the rhythm of the tides, and the trajectories of cannonballs could all be described, explained, and predicted by a small set of differential equations. ... Every inanimate thing in the universe bends to the rule of differential equations. I bet this is what [Richard] Feynman meant when he said that calculus is the language God talks. If anything deserves to be called the secret of the universe, calculus is it. (Strogatz, 2019)

Strogatz remarks highlight Newton’s impact on our own everyday understanding of physical laws: the differential, calculus-based framework Newton created fundamentally underlies our modern conception of the physical world.

His remarks also present an opportunity to make an important distinction.

Strogatz acknowledges that there are three distinct components to understanding: description, explanation, and prediction. In his *Principia*, Newton presents his Universal Law of Gravitation through the lens of description and prediction in the sense that he describes the gravitation as a force between two masses and he uses that description to predict the motion of the cosmos. However, while Strogatz may be interpreted as saying that differential equations provide causal explanations for the world, Newton did not hypothesize any such causes.

As we shall see, Newton articulated his laws in a *geometric* manner, describing the geometry of the motion of the cosmos, building on a tradition as old as Euclid himself. Calculus is not where Newton's intuition started. Newton began with geometry. In fact, we have a lecture, Richard Feynman gave on Newton's *Principia* where he gave a modern presentation of Newton's original demonstration of planetary motion. In his exposition in that lecture, Feynman says the following.

> In the beginning of our science—that is, in the time of Newton—the geometrical method of analysis in the historical tradition of Euclid was very much the way to do things. And as a matter of fact, Newton's Principia is written in a practically completely geometrical way—all the calculus things being done by making geometric diagrams. We do it now by writing analytic symbols on the blackboard, but for your entertainment and interest I want you to ride in a buggy for its elegance, instead of in a fancy automobile.

(Feynman qtd. in Goodstein and Goodstein, 1997)

While Einstein had highlighted the conceptual shift Newton achieved, Feynman illustrates how Newton initially expressed these concepts in a purely Euclidean style. Feynman's commentary also underscores the intellectual bridge between Greek geometry and Newton's laws of motion. Let us examine Feynman's explanation of Kepler's laws under Newtonian forces, presented in geometric form.

> We have to learn dynamics, we have to put them together. So now we have to explain what dynamics is all about.
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> What Newton means by this is this: that if this is the Sun, for instance, the center of the attraction, and at a given instant a particle were to, say, be here, and let me suppose that it moves to another point, from A to B in a certain interval of time. Then, [if] there were no forces acting toward the Sun, this particle would continue in the same direction and go exactly the same distance to a point c. But during this motion there's an impulse toward the Sun.
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> . . .
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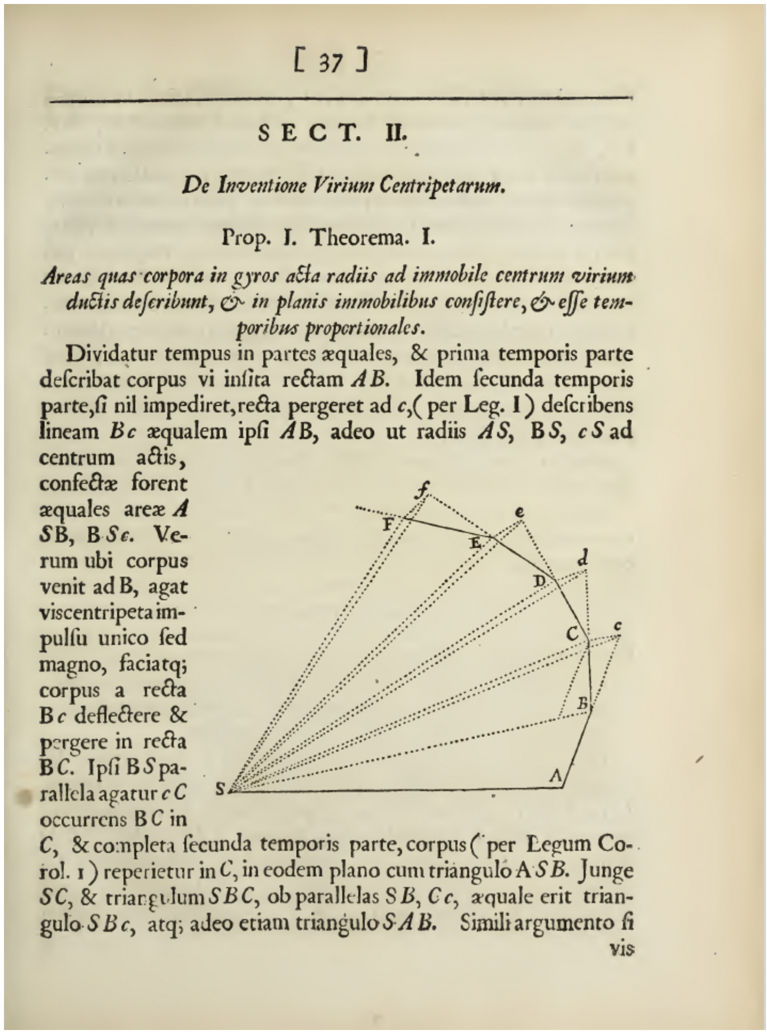
> And, therefore, the impulse is in the direction of the Sun, and this might represent the change in motion. That means that instead of this moving to here, it moves to a new point, which is C, which is different than c, because the ultimate motion is this motion



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SECT. II.

De Inventione Virium Centripetarum.

Prop. I. Theorema. I.

Areas quas corpora in gyros acta radiis ad immobile centrum virium ductis deſcribunt, & in planis immobilibus conſiſtere, & eſſe temporibus proportionales.

Dividatur tempus in partes æquales, & prima temporis parte deſcribat corpus vi inſita rectam *AB*. Idem ſecunda temporis parte, ſi nil impediret, recta pergeret ad *c*, (per Leg. I) deſcribens lineam *Bc* æqualem ipſi *A*B, adeo ut radiis *AS*, B*S*, *cS* ad centrum actis, confectæ forent æquales areæ *A S*B, B*Sc*. Verum ubi corpus venit ad B, agat vis centripeta impulſu unico ſed magno, faciatq; corpus a recta B*c* deflectere & pergere in recta B*C*. Ipſi B*S* parallela agatur *cC* occurrens B*C* in *C*, & completa ſecunda temporis parte, corpus (per Legum Corol. 1) reperietur in *C*, in eodem plano cum triangulo A*SB*. Junge *SC*, & triangulum *SBC*, ob parallelas S*B*, *Cc*, æquale erit triangulo *SBc*, atq; adeo etiam triangulo *SAB*. Simili argumento ſi
vis

Figure 1: The figure Feynman is referencing in his description of Newton's dynamics. This is the full page from the first edition of *Principia* held in the Boston Public Library and including the crucial figure.

> compounded from the original plus the additional impulse given toward the center of the Sun. So that the ultimate motion is along the line BC, and at the end of the second interval of moment of time the particle will be at C. (Feynman qtd. in Goodstein and Goodstein, 1997)

We see in Figure 1 that the change in motion (the differential step) is encoded in Euclidean geometry as the line segments cC, dD, eE, and fF. The proposition this diagram is in service immediately recalls Kepler's Second Law of Planetary Motion, which states that a line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time. The proposition reads:

> Areas quas corpora in gyros acta radiis ad immobile centrum virium ductis describunt, & in planis immobilibus consistere, & esse temporibus proportionales.
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> The areas, which revolving bodies describe by radii drawn to an immoveable centre of force, do lie in the same immoveable planes, and are proportional to the times in which they are described. Newton (1729)

The importance of Newton's reformulation of Kepler's Second Law in terms of his Laws of Motion in Figure 1 is that Newton gives an explanation for why Kepler's Second Law holds. He gives a geometric construction where Kepler's Second Law holds. Furthermore, the geometry represents a differential law, where the line segments cC, dD, eE, and fF represent a force towards an immovable point. The use of geometry to describe orbits was not new: Kepler, himself, first tried to describe orbits as spheres repeatedly inscribed and circumscribed in platonic solids, and then in his First Law of Planetary Motion as an ellipse American Physical Society (2014). Rather what was new was that Newton encoded in his geometric diagram the differential step—a change in the trajectory of motion–and identified a specific cause of the change in the step—a force towards an immovable center.

There are two sorts of intuition Newton exhibits in this example. There is his intuition in relation to constructing the geometry. Then, there is his intuition in relation to the differential law encoded in the geometry. We will consider both of these sorts of intuition throughout this paper.

Newton, early on in his studies at Trinity College, was well-trained in Euclid. One of his teachers was Isaac Barrow, who was author of the standard textbook on Euclid at the time (Swetz and Katz, 2011). Newton certainly received excellent instruction in Euclid at least through attendance of Barrow's lectures. We know that Newton received a fellowship at Trinity College and the Lucasian professorship due to Barrow's aid (Finegold, 1993). Barrow in a preface to a publication of his own lectures on geometry, writes that he "thanks Newton—'a Man of great Learning and Sagacity'—for helping to revise the text" (Thomas, 2018, p. 71).

In giving the original, Euclidean presentation of Newtonian physics, Feynman echoes what Bertrand Russell wrote about the *Principia* in his *The History of Western Philosophy*: namely, that "The form of Newton's *Principia*, in spite of its admittedly empirical material, is entirely dominated by Euclid."

Though Newton's Universal Law of Gravitation was stated and presented using Euclidean geometry, geometry does not wholly capture Newton's ideas. Geometry is static in the sense that, while geometry has clear notions of length, geometry does not have clear notions of time. The Greeks describe the motion of a planet by the geometry of the trajectory they follow—the trajectory is static. Newton describes the motion of the planet by the forces that are acting on it over time—the trajectory is given dynamically. Russell writes:

> Greek astronomy was geometrical, not dynamic. The ancient thought of the motions of the heavenly bodies as uniform and circular, or compounded of circular motions. They had not the conception of *force*. There were spheres which moved as a whole, and on which the various heavenly bodies were fixed. With Newton and gravitation a new point of view, less geometrical, was introduced. It is curious to observe that there is a reversion to the geometrical point of view in Einstein's General Theory of Relativity, from which the conception of the force, in the Newtonian sense, has been banished (Russell, 1945, p. 216)

We now begin to see why, given that the Greek geometers equipped with the same tools of geometry as Newton, did not discover the Universal Law of Gravitation for themselves. Newton, uniquely, had the concept for the 'differential law,' expressing the world as dynamic and not static. Newton's *Principia* is similar in *form* to Greek geometry in form in the sense that both the *Principia* and Greek geometry use axiomatic, deductive reasoning, outlining definitions and propositions. But is is dissimilar in terms of dynamism.

Dynamic, as we see from the discussions above, means that there are forces at play. The concept of force is inherently tied to the concept of the differential; the application of a force imparts an impulse which *causes* a differential, a change in motion. Euclidean geometry, as Newton illustrated in the *Principia* and as we walked through following Feynman's lead (Figure 1), is capable of encoding dynamic forces within its method. Though dynamism is available within the mathematics of geometry created by the Greeks, dynamism is not used in geometry as practiced by the Greeks. Something outside of the mathematics, perhaps as we will contend a philosophical element related to the Aristotelian conception of causation, precludes dynamism from geometry as practiced by the Greeks.

In relation to our goal of understanding Newton's own intuition, we will take a non-digression digression to establish to some degree Newton's familiarity with Aristotle. Regarding Newton's education in Greek philosophy, James Gleick recounts in his biography, *Isaac Newton*, the following.

> The curriculum [at the University of Cambridge] had grown stagnant. It followed the scholastic tradition laid down in the university's medieval beginnings ... The single authority in all the realms of secular knowledge was Aristotle—doctor's son, student of Plato, and collector of books. Logic, ethics, and rhetoric were all his, and so—to the extent they were studied at all—were cosmology and mechanics. ... Supplemented by ancient poets and medieval di-

> vines, it was a complete education, which scarcely changed from generation to generation. Newton began by reading closely, but not finishing, the *Organon* and the *Nicomachenan Ethics*. (Gleick, 2004, p. 22)

Gleick discerned Newton's close reading of the *Organon* and the *Nicomachenan Ethics* from Newton's 'Trinity College Notebook', which he kept as an undergraduate, among others of Newton's notebooks, books, and papers. Newton, as a boy, had been taught Latin, Hebrew, and Greek (Gleick, 2004, p. 13). In his notes on Aristotle for his undergraduate studies, Newton made evident his facility of Latin and Greek.

> On folios 3r-10v Newton made notes based on Aristotle's Organon and Porphyry's Isagoge. These are probably the earliest in the notebook [Quæstiones quædam Philosophiæ]. In a youthful hand, they are from a four-volume edition of Aristotle's works entitled Aristotelis Opera omnia, quae extant, Graece et Latine, third edition (Paris, 1654), which was under the general editorship of Guillaume Du Val. There is a copy of this work in the Cambridge University Library and two in the British Library.
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> Although the notes are in Greek, Newton maintained running headings in Latin in the margins.
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> . . .

> Of Aristotle's complete list of categories, Newton covers substance, quantity, relation, quality, and passion. (McGuire and Tamny, 1983, p. 15)

Here is what Aristotle has to say in his category of quantities that Newton took notes on. We see in the Aristotelian conception of the continuity of space and time some similarities but also stark differences to the Newtonian differential and integral calculus conception of continuity of space and time.

> Space and time also belong to this class of quantities. Time, past, present, and future, forms a continuous whole. Space, likewise, is a continuous quantity; for the parts of a solid occupy a certain space, and these have a common boundary; it follows that the parts of space also, which are occupied by the parts of the solid, have the same common boundary as the parts of the solid. Thus, not only time, but space also, is a continuous quantity, for its parts have a common boundary. Aristotle, 1.1.6

Modern calculus and physics have at its heart concepts of continuity. In physics, it is very sensible to say that we integrate a force over continuous time to calculate the impulse or that we integrate a force over continuous distance to calculate the work. In Newton's publication of his physics in the *Principia*, he would write that "Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external."

However, as we have reviewed, Newton's description of force in the geometrical diagrams he presented (Figure 1) time has been discretized into steps, whereby the gravitational force can act in discrete impulses. There is an apparent contradiction where Newton appears to suggest that time is both discrete and continuous. Of course, Newton's resolution to this question is to say that the geometrical picture is an approximation of reality and, in taking smaller and smaller time steps, we more closely reach reality.

As Einstein remarked, it is Newton's "clear conception of the differential law", precisely describing the geometry of the differential step and solving cosmological motion using this geometry, that was such an important achievement. We might ask: Why did it take more than two millennia from when Euclid described his geometry to when Newton used it to demonstrate the physics of the cosmos? Given that Aristotle also viewed time as a continuous quantity, we might expect that Aristotelians would inquire as to the mechanics of how to describe nature transitioning from one moment in time to the next. In brief, the primary reason is that while Newton in his mechanics, is interested in the past causes the present, Aristotle in his physics and metaphysics is interested in how the past and the future causes the present. Aristotelians did not believe that physics could be described in terms of mechanics, so they would not embark on Newton's program and the study of the cosmos from Aristotle to Newton was firmly Aristotelian.

We will describe why Aristotle believed this by considering his view of causation and then we will consider the Greek Atomists, who were the "great materialists of antiquity" (Burnet, 1908, p. 389).

Aristotle introduced four causes to understand why things are the way they are. The material cause identifies the matter something is made from—like bronze for a statue. The formal cause is the form or essence that makes it what it is. The efficient cause is the agent or process that brings it about, like a sculptor carving marble. Finally, the final cause is the purpose or end—the reason why something exists or occurs.

Among these, Aristotle placed particular emphasis on the final cause. He believed that natural processes tend toward certain ends. Living things, for example, act with purpose. Aristotle notes that "the living creature is moved by intellect, imagination, purpose, wish, and appetite. And all these are reducible to mind and desire" (Aristotle, 350 B.C.E, part 6). Russell illustrates this with a dog seizing a bone: the dog's action is guided by its desire for the bone and the satisfaction it brings, not just by mechanical impulses.

For Aristotle, the final cause had priority in explaining phenomena. Knowing how something happened—its material and efficient causes—is incomplete without understanding why it happened—its final cause. The final cause gives actions context and meaning.

From an Aristotelian perspective, understanding celestial motion cannot rely on mechanics alone. Without acknowledging a guiding final cause, there would be little reason for an Aristotelian to embark on the purely mechanistic analysis that characterized Newton's approach. Given the dominance of Aristotelian thinking prior to Newton, it is perhaps unsurprising that, given few people would find reason to embark on a purely mechanistic analysis, no one would succeed in a mechanical characterization of the motion of the planets as Newton does.

This view was not unfamiliar to Newton. Let us return to Isaac Barrow.

> During the early 1650s Barrow read widely. He was well versed in Aristotle and retained a deep respect for Aristotle throughout his career, describing him as the 'unchallenged Prince of all who have ever been or ever will be philosophers' (N IX 161). During this period, Barrow also engaged with René Descartes in depth. This early enthusiasm is displayed in his 1652 *Cartesiana Hypothesis De Materia et Motu Haud Satisfacit Praecipuis Naturae Phaenomenis*. The essay opens by discussing the post-Aristotelian neglect of natural philosophy, and goes on to praise Descartes—that best and most ingenious (ingeniosissimus) philosopher—and his natural philosophy in high terms (N IX, 79–81). Despite this praise, Barrow does not accept all aspects of his work. . . . Barrow is wary of the materialist connotations of Descartes' system, writing that it exhibits no vital spirit (Spiritum quendam vitalem) (N IX, 82–104). . . . Barrow later grew more critical of Descartes, likely because Barrow also perceived Cartesianism to carry the risk of materialism and atheism. (Thomas, 2018, p. 69)

Barrow's concern with the 'vital spirit' is in line with Aristotle's concern for final causes. The 'vital spirit' that Barrow desires is plainly the very same 'living creature's view' that Einstein describes as the subject of Newton's quest to excise.

That is not to say, however, the mechanical view of physics, which does not attribute motion to soul, will, or desires, is intuitive. As Einstein recites, the view that "all material events should be traced back to a strictly regular series of movements" has been present since the time of the Greeks. The Greeks who espoused this view were called Atomists. Let us more fully consider how an Atomist would explain motion. Interpreting for us how the Greek Atomists Leucippus and Democritus would answer a question on what the cause of motion is, Cyrus Bailey writes the following.

> Leucippus seems to have asserted in a rather half-hearted manner that 'necessity' was the motive cause, and that by his assertion he intended not, as previous thinkers have done, to introduce an inexplicable external force to explain what could not otherwise be shown to follow from his fundamental principles, but rather to adumbrate the idea familiar to us as that of 'natural law'. The ultimate controlling principle is that everything follows the laws of its own being. This notion which Leucippus applied with some hesitation to explain the original motion of the atoms, Democritus now confidently asserts with a much wider, indeed, a universal application. 'Necessity' orders all things, indeed by necessity the whole course of things is foreordained from all eternity; the whole history of the universe is but the inevitable outcome, step by step, of its original and eternal constitution. (Bailey, 1964, p. 121)

Democritus's conception of 'necessity' as that of the "step by step" application of "its [an atom's] original and eternal constitution" is akin to Newton's differential law, where at each step, the forces of natural law are applied. The comparison in intuition between the Atomists and Newton is striking because of their commonality across great stretches of history. However, nearly by definition, these intuitions that reflect similarities in views between the Atomists and Newton cannot serve to distinguish what makes Newton's intuition special. The precedent for that intuition makes Newton's possession of that intuition less exceptional.

Having given discussion of the Atomists whom we will return to later, let us tackle Aristotle's *Metaphysics*. This discussion will be somewhat lengthy, but is in service of establishing what intuitions Aristotle and thus Ptolemy and the Catholic Church and the Anglican Church and thus Isaac Barrow (who was Anglican and an Aristotelian) held, and what intuitions Newton would have to struggle against.

We first pick up on Aristotle's *Metaphysics* with his conception of motion and we will accept, as he explains, that time is infinite. Proceeding from that premise, he writes.

> It is impossible that movement should either have come into being or cease to be (for it must always have existed), or that time should. For there could not be a before and an after if time did not exist. Movement also is continuous, then, in the sense in which time is; for time is either the same thing as movement or an attribute of movement. And there is no continuous movement except movement in place, and of this only that which is circular is continuous. (Aristotle, 350 B.C.E., XII.6)

Aristotle's intuition is that "there is no continuous movement except movement in place, and of this only that which is circular is continuous". We say that it is his intuition because in *Metaphysics* he gives no explanation for both claims. We can try to explore his intuition, though, and guess at why he might hold it and where it does not match up with the Newtonian and Euclidean view of space and geometry.

For the first claim that “there is no continuous movement except movement in place”, we can reach the same claim that Aristotle intuits if we are to assume that space is finite in extent, though we might express it differently. We might say that ‘continuous movement in a finite space is bounded,’ which is certainly true because it is finite. This explanation of the first claim is not altogether satisfying, though perhaps its simplicity is why Aristotle provided no explanation other than his bare statement.

His second claim that “of [movement in place] only that which is circular is continuous” is also a bit shocking. As an arbitrary example, going around in an ellipse would be continuous movement in place. We might interpret him as saying instead that the only movement in place is a cycle, e.g. going around and returning back to the exact same place. This seems—intuitively—to be quite reasonable: how can you go around in the same place without at some point repeating a pattern? Yet, Edward Lorenz with his strange attractor showed that it is in fact possible to go around and around in the sample place without repeating a pattern (Figure 2).

In his work *On the Heavens*, Aristotle provides a more complete argument.

> This circular motion is necessarily primary. For the perfect is naturally prior to the imperfect, and the circle is a perfect thing. This cannot be said of any straight line:-not of an infinite line; for, if it were perfect, it would have a limit and an end: nor of any finite

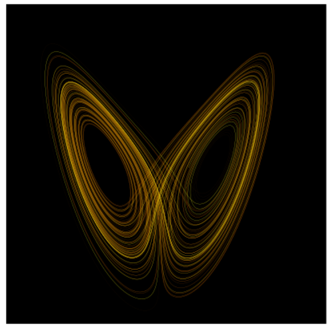


Figure 2: Reproduced from Wikipedia contributors (n.d.)

> line; for in every case there is something beyond it, since any finite line can be extended. And so, since the prior movement belongs to the body which naturally prior, and circular movement is prior to straight, and movement in a straight line belongs to simple bodies-fire moving straight upward and earthy bodies straight downward towards the centre-since this is so, it follows that circular movement also must be the movement of some simple body.

This argument in its appeal to the perfection of the circle has a Platonic nature and in its absence of an appeal to continuity, which has precise, modern, mathematical definitions, is much more resistant to analysis. We know—from Newton's demonstration—that the Aristotelian conception of the circular movement as primary is incorrect in favor of the Newtonian conception of linear movement in an infinite line as primary.

As an observation, Euclid's first postulate is that "A straight line can be drawn between any two points", his second postulate is that "A straight line can be extended indefinitely in a straight line" and his third postulate is that "A circle can be drawn with any center and radius." The first postulate assuredly must come prior to the second postulate, because the infinite line is defined in terms of the finite. The first postulate likely must come prior to the third because radius invokes the concept of distance which seems to require the concept of a finite line. This is a reversal of Aristotle's ordering! We see again that the intuitions of Newton and Aristotle seem to be opposing in some sense and this is a fundamental question indeed. Newton embodies this in his First Law of Motion. Again, however, we see that Newton's intuition is predated by millennia by Euclid. Connecting Newton's intuition temporally mirrors the connection we continually make between Newton and the Atomists, indicating these are strong intuitions, indeed. Admittedly, the connection to Euclid is, though interesting, arguably tenuous, so we also note that Galileo formulated a nearly equivalent inertial law, which was later supported philosophically by Descartes and empirically by Pierre Gassendi, before Newton's time Drake (1964).

Setting the primacy (or lack thereof) of lines and circles considerations aside, supposing that, as Aristotle suggests, circles have primacy, let us continue with his argument.

> The final cause, then, produces motion as being loved, but all other things move by being moved. Now if something is moved it is capable of being otherwise than as it is. Therefore if its actuality is the primary form of spatial motion, then in so far as it is subject to change, in this respect it is capable of being otherwise,-in place, even if not in substance. But since there is something which moves while itself unmoved, existing actually, this can in no way be oth-

> erwise than as it is. For motion in space is the first of the kinds of change, and motion in a circle the first kind of spatial motion; and this the first mover produces. (Aristotle, 350 B.C.E., XII)

In Aristotle's view, stars and planets move in circles because they love the the first mover. The first mover is the final cause of the movement of stars and planets. Intuitively, extending the previous analogy from Russell, the first mover is like the bone and the stars and planets are like a dog desiring the bone. Aristotle continues on to describe how the best of Greek astronomers and mathematicians, Eudoxus and Callippus, describe the movement of stars and planets in terms of spheres. He then observes, preempting Ptolemy, the necessity for epicycles.

> It is necessary, if all the spheres combined are to explain the observed facts, that for each of the planets there should be other spheres (one fewer than those hitherto assigned) which counteract those already mentioned and bring back to the same position the outermost sphere of the star which in each case is situated below the star in question; for only thus can all the forces at work produce the observed motion of the planets. (Aristotle, 350 B.C.E., XII)

We see contained within the above passage similar ideas of Newton in a sense. We might selectively quote Aristotle to say that 'all the forces at work produce the observed motion of the planets'. It would be surprising to see

Aristotle directly mentioning forces, so consulting a different translation, we see that force is not mentioned—rather motion is considered.

> καθ᾽ ἕκαστον τῶν πλανωμένων ἑτέρας σφαίρας μιᾷ ἐλάττονας εἶναι τὰς ἀνελιττούσας καὶ εἰς τὸ αὐτὸ ἀποκαθιστάσας τῇ θέσει τὴν πρώτην σφαῖραν ἀεὶ τοῦ ὑποκάτω τεταγμένου ἄστρου: οὕτω γὰρ μόνως ἐνδέχεται τὴν τῶν πλανήτων φορὰν ἅπαντα ποιεῖσθαι.
>
> there must be for each of the other planets other spheres, one less in number than those already mentioned, which counteract these and restore to the same position the first sphere of the star which in each case is next in order below. (Aristotle, 1924, XII)

A more faithful reading of Aristotle would be that Aristotle says that it is the circular motion which in combination produces the observed motion of the planets—motion itself in combination, not forces.

We take from this several points.

The first is that Newton's intuitions achieved a total victory.It is now so intuitive that forces are the cause of motion in the cosmos that we naturally slip into talk of forces when discussing the cause of cosmological motion. The former translation mentioning forces was taken from William David Ross—who is described as the "the most influential Aristotelians of the twentieth century. ... [His translations] are still held in high regard" Skelton (2022). The second is that the intuitions of Aristotle as rendered in his *Metaphysics* were sufficiently close to to Newton's intuitions that translating Aristotle as saying "all the forces at work produce the observed motion of the planets" is in fact a good translation. The practical distance in their intuitions is perhaps smaller than commonly described. The third is that Aristotle's intuitions were so nearly correct in predicting the motion of the planets. If Aristotle or Ptolemy had the data and the computational faculties, they could have, similarly to Newton, precisely approximated the motion of the planets and the stars, because epicycles can express the (precisely) Fourier-approximated motion of the planets and the stars Acosta et al. (2019).

In totality, we have described how closely aligned Newton's and Aristotle's intuitions were, except that while Newton approximated the curve of planets orbit with a line, Aristotle approximated the curve of planets with circles. In some sense, the difference in physical intuition between Aristotle and Newton is in the primacy of the circle or the line. In the Newtonian conception of space, the infinite line is the continuous motion. In the Aristotelian conception of space, the circle is the continuous motion.

Newtonian geometry, as we have discussed, is inherently Euclidean geometry. Euclidean geometry came after Aristotle's work and dominated subsequently to the time of Newton and then was carried by Newton's system up until Einstein. Einstein, in fact, showed that space in the universe is non-Euclidean. Einstein writes:

> We have nevertheless to admit today that our position in regard to the fundamental laws of this motion resembles that of astronomers before Newton in regard to the motions of the planets.

In Einstein's Theory of general relativity, it is spacetime itself which is warped and it is the deformation in spacetime due to mass that gives the

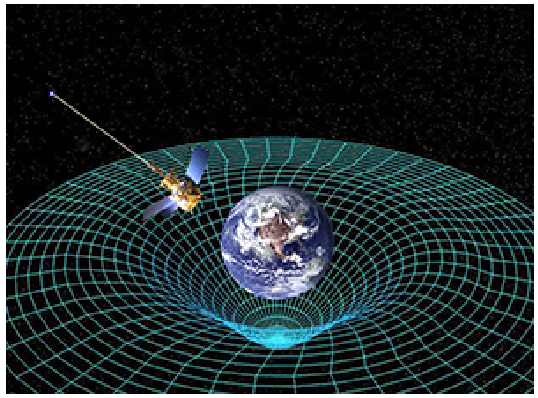


Figure 3: NASA's depiction of the 'straight lines' through spacetime, which are called geodesics, in *Physics of the Cosmos* of the Cosmos Program (2024).

phenomenon of gravity. The Law of Universal Gravitation existing in Euclidean geometry, as given by Newton, is a (very good) approximation of the warping of spacetime for our solar system and beyond, but an approximation nonetheless. In Euclidean geometry, continuous motion can be though of, in the Newtonian sense, to be motion in a straight line. For Newton, gravity is a force operating within this geometry.

Einstein's description of spacetime is different. The force of gravity under general relativity is not a force acting in Euclidean geometry. Rather, gravity is represented by the warping of spacetime in a non-Euclidean geometry. Due to warping, a continuous motion through spacetime can become a circular orbit. For example, the moon traveling through a 'straight line' through the non-Euclidean space described by Einstein's general relativity appears to us to be the moon traveling in an orbit around the Earth.

It is quite remarkable the Aristotle intuitions on motion have survived Newton's revolution in mechanics and made a comeback with Einstein's general relativity. Though the relationship between Aristotle's thought and general relativity is generally outside our considerations here, we simply use the occasion to note that for some of the trickiest intuitions we've considered in this paper–whether continuous motion is fundamentally that of a circle or a line–there hasn't been a clear, correct answer. It's not clear what it would mean for one intuition to be correct or not. Newton's is surely more useful for the industrial and scientific revolutions, and Newton's is the foundation of physics to date, but Einstein's is our most advanced theory of physics. That is to say, it is not clear that we can attribute any single intuition as being Newton's exceptional intuition.

Einstein points us to 'clear conception of the differential law' as one of Newton's greatest achievements. When considering what is the most modern and most precise theory of the physics of the planets and the stars, it is not Newton's Universal Law of Gravitation or Newton's demonstrations in Euclidean geometry that persists. Rather it is Newton's 'clear conception of the differential law' that persists, embodied in the differential equations that govern general relativity. It is not even Newton's notation—Gottfried Wilhelm Leibniz's notation has become more popular—which has persisted. Rather it is Newton's 'clear conception of the differential law'. Einstein could have equivalently said that is Newton's clear intuitive grasp of the differential law, which is one of Newton's greatest intellectual achievements.

Feynman writes, in the same lecture on Newton's Euclidean methods for describing gravitation, the following.

> It is not easy to use the geometrical method to discover things. It is very difficult, but the elegance of the demonstrations after the

> discoveries are made is really very great. The power of the analytic method is that it is much easier to discover things than to prove things.

Feynman is certainly a proponent of a visual method of demonstration, having formulated his eponymous diagram. Feynman's characterization of the geometrical versus the analytic methods mirrors the structure and import of the three books of Newton's *Principia*. In the following discussion, facts about the contents and contemporary reception of the *Principia* in the following discussion are taken from the Smith and Iliffe (2024) entry in *The Stanford Encyclopedia of Philosophy*.

The opening sections of the *Principia* give definitions and Newton's Laws of Motion. The first book of *Principia* is, as Russell noted and we've seen in Feynman's exposition, is Euclidean in form and gives his presentation of centripetal motion—notably including, the differential and his proofs relating the inverse square laws to Kepler's laws. The second book of *Principia* aims to refute Descartes and Leibniz's view of literal fluid vortices as maintaining orbits. We say it aims to refute because it is incorrect or otherwise unsuccessful across multiple dimensions (though the Newton assuredly arrives at the correct conclusion that gravity is not due to the fluid vortices Descartes and Leibniz describe). The third book of *Principia* derives his Law of Universal Gravitation and then goes on to calculate various astronomical quantities.

We will concern ourselves with the first and third book of *Principia* for those are the ones with lasting import. The first book, in its Euclidean form, yields a geometrical picture of the differential characterization of Kepler's Second Law. The figure shows how motion is composed of differential steps. We recall that this picture is the same as the one Feynman described for us in Figure 1.

It is of this geometrical picture in the first book, among others, that a reviewer gave the following remarks, which a modern reader might find puzzling.

> The work of M. Newton is a mechanics, the most perfect that one could imagine, as it is not possible to make demonstrations more precise or more exact than those he gives in the first two books.... But one has to confess that one cannot regard these demonstrations otherwise than as only mechanical; indeed the author recognizes himself at the end of page four and the beginning of page five that he has not considered their Principles as a Physicist, but as a mere Geometer....
>
> In order to make an opus as perfect as possible, M. Newton has only to give us a Physics as exact as his Mechanics. He will give it when he substitutes true motions for those that he has supposed (*A review of Principia in the Journal des Sçavants quoted by* Smith and Iliffe (2024)).

First, we see that the reviewer confirms previous assertions that the geometric demonstration is most elegant (this is not the puzzling part). The reviewer effusively remarks that "it is not possible to make demonstrations more perfect or more exact." The puzzling part is that, second, we notice that the reviewer suggests that Mechanics is separate from Physics and that Newton, having posed his Laws of Motion, "has not considered their Principles as a Physicist, but as a mere Geometer." By transitivity, the reviewer would call entire swathes of modern physics, geometry.

A modern physics textbook, contains passages like the following from *University Physics*, considering mechanics as a part of physics.

> Mechanics is the study of the relationships among force, matter, and motion. In this chapter and the next we'll study kinematics, the part of mechanics that enables us to describe motion. Later we'll study dynamics, which helps us understand why objects move in different ways. (Young and Freedman, 2019, p. 22)

A modern physics textbook like *University Physics* will have headings such as "Mechanics" and "Thermodynamics" and "Newton's Laws of Motion" and "Fluid Mechanics" and "Quantum Mechanics". All of these would have been describe the reviewer—and likely many others in his time—would have described as not physics.

Newton's intuition was that mechanics is, in fact, physics. Mechanics, in Newton's conception as it is today, is the study of the application of forces and the motion produced by forces. Yet, it was not obvious—or, perhaps we should say, intuitive—in Newton's time that the cause of motion is forces. It was not intuitive that describing the mechanics of a system, the differential law by which the system evolves, amounts to describing the physics of a system.

In Aristotle's conception of his four causes, he had the efficient cause ('Where does change (or motion) come from?') and the final cause ('What is its good?'). He could admit that the efficient cause—the forces—could be the cause in some sense, but that the final cause is the 'real cause'. For Aristotle, the explanatorily prior cause was the final cause and the motion of the astronomical objects was towards the final cause. In Aristotle's consideration of the stars there was still a divine character—or at least a metaphysically prior character—that by definition freed the heavens from the efficient cause and placed the heavens squarely in the dominion of the final cause. Physics had a vivacious quality and was infused by desire, soul, and meaning.

Newton took a mechanistic view. As Einstein described, Newton formulated his 'differential law' and mechanistically explained the cosmos. Newton's intuition was to treat the sun, stars, planets, and comets all as matter with mass and to write his differential laws of motion and gravitation, which governed mass. His intuition, therefore, was not only to conceive the mechanical view, but also to apply the mechanical view to the cosmos as the immediate cause. In doing so, he stripped out notions of final causes in astronomical considerations, and in doing so clarified the matter greatly.

The mechanical intuition Newton demonstrated, continuing the spirit of the Atomists, would come to drive modern physics. However, in relation to Newton's intuition, just as he wasn't unique in adopting the mechanical view, he also wasn't unique in applying the mechanical view to the cosmos for Atomists did the same. Anaxagoras was found guilty of a crime for teaching the mechanical view of the cosmos (Russell, 1945, p. 62). Anaxagoras taught, according to philosopher Jonathan Barnes in his book *Early Greek Philosophy*, the following:

> He said that the sun is a fiery lump, larger than the Peloponnese

> (but some ascribe this to Tantalus), and that the moon is inhabited and also contains hills and ravines. The uniform stuffs are first principles; for just as gold is compounded from gold-dust, so the universe is combined from small uniform bodies. (Barnes, 1987, p. 237)

Anaxagoras' views were widely circulated, in large part through Aristotle's considerations of Anaxagoras in his view, though it is difficult to say with certainty what Newton knew of it.

Regardless, though, the important consequence of considering Anaxagoras's views and the view of other Atomists is that we can confidently say that Newton's mechanical view of the cosmos was not unique. There were others with the same intuition.

This is sensible, because it is difficult to have firm intuitions about the cosmos, which Newton and the Greeks can only observe but have no direct or clear experience of. Many multitudes of intuitions can be somewhat reasonable.

For Newton, at the heart of his mechanical intuition was a geometric picture—a Euclidean picture. The reviewer was correct in that he geometric picture was, in a sense, mechanical for it was the repeated application of axiomatic, deductive logic. The picture was confirmed through calculation by the agreement it produced with known patterns of motion in the stars. Newton's ability to calculate pervades throughout his endeavors, particularly his analytic calculations in Book 3 of *Principia* to calculate all sorts of astronomical phenomena.

In a letter to Richard Bentley who would be in charge of the second edition of *Principia* and who gave a lecture on *A Confutation of Atheism* the same year of the letter, Newton writes the following.

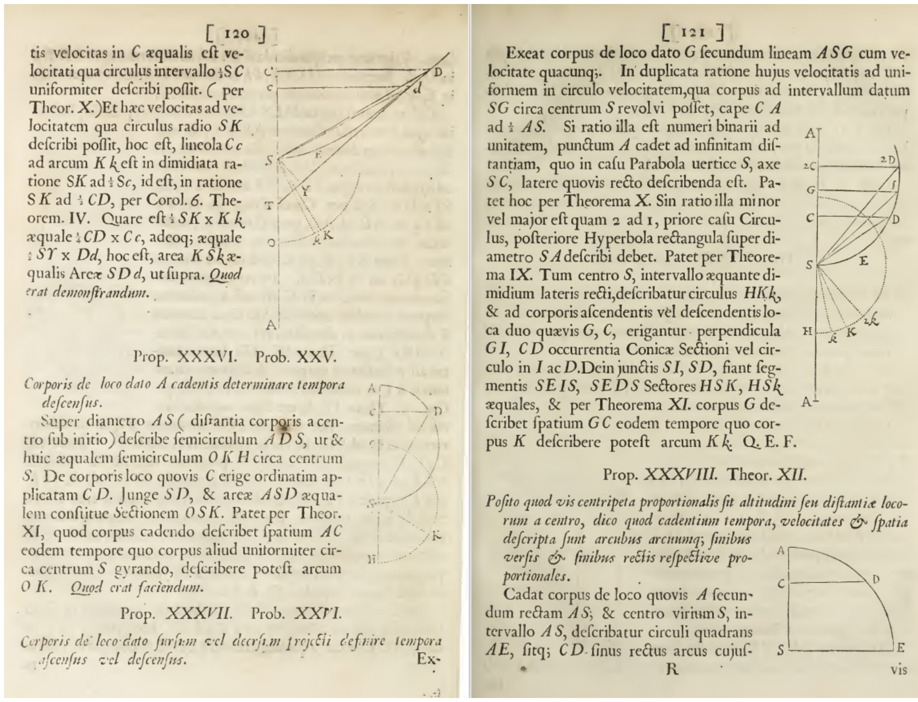
> When I wrote my treatise about our Systeme [Principia] I had an eye upon such Principles as might work with considering men for the Beleife of a Deity & nothing can rejoyce me more then to find it usefull for that purpose. But if I have done y^{e} publick any service this way 'tis due do nothing but industry & a patient thought. (Newton, 1967, p. 9)

It is quite remarkable that Newton would attribute the contributions of the *Principia* to "nothing but industry & a patient thought." There is surely an element of modesty to his statement, especially considering the theological backdrop of the letter, but there likely is an element of truth as well. It is hard to see how intricate work like this (Figure 4) could be the product of anything but patient thought.

His patient thought extends beyond the geometric. We can see from his notebooks patient thought regarding the formation of his Laws of Motion. He goes on and on enumerating axiomatic statements until he arrives at his Three Laws of Motion we see in *Principia* (Figure 5).

For example, here are two of his axioms, numbered 103 and 104.

> 103 By the same reason alsoe If two bodys rest or bee equivelox: then as the body (a) is to the body (b) soe must the power or efficacy vigor strength or virtue of the cause which begets new velocity in (a) bee to the power virtue or efficacy of the cause



[120]

tis velocitas in C æqualis eſt velocitati qua circulus intervallo ½ SC uniformiter deſcribi poſſit. (per Theor. X.) Et hæc velocitas ad velocitatem qua circulus radio SK deſcribi poſſit, hoc eſt, lineola Cc ad arcum Kk eſt in dimidiata ratione SK ad ½ Sc, id eſt, in ratione SK ad ½ CD, per Corol. 6. Theorem. IV. Quare eſt ½ SK x Kk æquale ½ CD x Cc, adeoq; æquale ½ ST x Dd, hoc eſt, area KSk æqualis Areæ SDd, ut ſupra. *Quod erat demonſtrandum.*

Prop. XXXVI. Prob. XXV.

Corporis de loco dato A cadentis determinare tempora deſcenſus.

Super diametro AS (diſtantia corporis a centro ſub initio) deſcribe ſemicirculum ADS, ut & huic æqualem ſemicirculum OKH circa centrum S. De corporis loco quovis C erige ordinatim applicatam CD. Junge SD, & areæ ASD æqualem conſtitue Sectionem OSK. Patet per Theor. XI, quod corpus cadendo deſcribet ſpatium AC eodem tempore quo corpus aliud uniformiter circa centrum S gyrando, deſcribere poteſt arcum OK. *Quod erat faciendum.*

Prop. XXXVII. Prob. XXVI.

Corporis de loco dato ſurſum vel deorſum projecti definire tempora aſcenſus vel deſcenſus.

Ex-

[121]

Exeat corpus de loco dato G ſecundum lineam ASG cum velocitate quacunq;. In duplicata ratione hujus velocitatis ad uniformem in circulo velocitatem, qua corpus ad intervallum datum SG circa centrum S revolvi poſſet, cape CA ad ½ AS. Si ratio illa eſt numeri binarii ad unitatem, punctum A cadet ad infinitam diſtantiam, quo in caſu Parabola uertice S, axe SC, latere quovis recto deſcribenda eſt. Patet hoc per Theorema X. Sin ratio illa minor vel major eſt quam 2 ad 1, priore caſu Circulus, poſteriore Hyperbola rectangula ſuper diametro SA deſcribi debet. Patet per Theorema IX. Tum centro S, intervallo æquante dimidium lateris recti, deſcribatur circulus HKk, & ad corporis aſcendentis vel deſcendentis loca duo quævis G, C, erigantur perpendicula GI, CD occurrentia Conicæ Sectioni vel circulo in I ac D. Dein junctis SI, SD, fiant ſegmentis SEIS, SEDS Sectores HSK, HSk æquales, & per Theorema XI. corpus G deſcribet ſpatium GC eodem tempore quo corpus K deſcribere poteſt arcum Kk. Q. E. F.

Prop. XXXVIII. Theor. XII.

Poſito quod vis centripeta proportionalis ſit altitudini ſeu diſtantiæ locorum a centro, dico quod cadentium tempora, velocitates & ſpatia deſcripta ſunt arcubus arcuumq; ſinibus verſis & ſinibus rectis reſpective proportionales.

Cadat corpus de loco quovis A ſecundum rectam AS; & centro virium S, intervallo AS, deſcribatur circuli quadrans AE, ſitq; CD ſinus rectus arcus cujuſ-

R vis

Figure 4: Pages from the first edition of *Principia* held in the Boston Public Library.

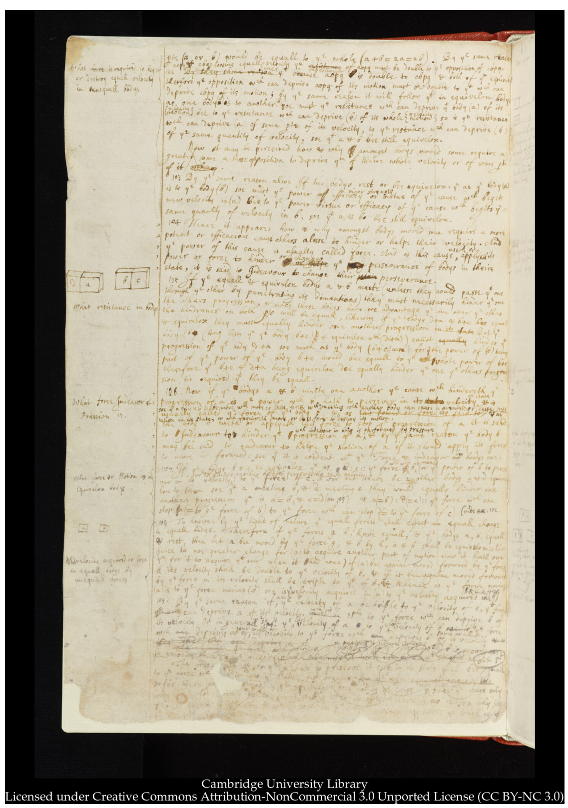


Figure 5: Page 12v of Newton's Waste Book (MS Add. 4004) Newton (c. 1612–c. 1690)

> which begets the = same quantity of velocity in b , soe that a & b bee still equivelox.
>
> 104 Hence it appeares how & why amongst bodys moved some require a more potent or efficacious cause others a lesse to hinder or helpe their velocity. And the power of this cause is usually called force. And as this cause useth or applyeth its power or force to hinder or change the perseverance of bodys in theire state, it is said to Indeavour to change their perseverance. (Newton, 1664–1685, p. 12v)

About Axiom 103, Gleick writes: we see how "Writing in English, he was constrained by the language at hand. At times his frustration was palpable in the stream of words . . . *Power efficacy vigor strength virture*—something was missing." Furthermore in Axiom 104, we see a hesitation in language when Newton writes that "the power of this cause is *usually* [empahsis added] called force." Of course, we call it force today, but a cloud over language hung over Newton when he crafted a wholly new system of and language for physics. Newton would continue on to Axiom 120 on the following page.

It is clear from the (at minimum) 120 axioms that would eventually be condensed down to his Three Laws of Motion that there wasn't a flash of insight that immediately showed him an answer. There was certainly some insight, and then continued patient application of thought.

What was special about Newton's intuition then? We have seen repeatedly our considerations how Newton's intuitions were connected to those of others including the Greek thinkers. We have seen how Newton's intuitions have been powerful and similar, at least to a few great persons of history, in that sense his intuitions haven't been wholly unique.

His distinguished power came from, as he himself suggests, patient thought. Newton worked many times in isolation—isolated at his farm during the plague and removed to Cambridge from London, away from the day-to-day and the exchanges at the Royal Society.

His powers of intuition are realized not from any single axiom or demonstration or experience, but rather the patient consideration and improvement of each one. His powers of intuition are not the starting point from which straightforward reasoning follows. Rather, his powers of intuition are realized at each step of of his reasoning. In this sense, he has a powerful meta-intuition.

Of course, it helps that in his patient thought, he applies this powerful meta-intuition many times.

Locke in his *An Essay Concerning Human Understanding* uses Newton to illustrate demonstrative knowledge:

> Nobody, I think, can deny, that Mr. Newton certainly knows any proposition that he now at any time reads in his book [Principia] to be true; though he has not in actual view that admirable chain of intermediate ideas whereby he at first discovered it to be true. (Locke, 1690, IV.i.9)

Locke, notably in close private correspondence with Newton, makes the sensible claim that Newton's propositions in the *Principia* are not on their own a stroke of genius but rather of an "admirable chain of intermediate ideas"—

we may take "admirable" to refer to Newton's power of meta-intuition, the intuition he possesses to make intelligent steps of reasoning.

When Locke refers to Newton's propositions, he is thinking quintessentially of propositions in the Euclidean sense. This is made all the more evident when he writes that in demonstrative knowledge, "Demonstration depends on clearly perceived proofs" (IV.2.3). By virtue of presenting his geometric picture by using Euclidean proof, Newton satisfies Locke's condition for demonstrative knowledge, where truths are perceived "by bare intuition" without further intermediaries (IV.2.1). We recall that in Feynman's lecture on the geometry of the *Principia* he said, "It is not easy to use the geometrical method to discover things. It is very difficult, but the elegance of the demonstrations after the discoveries are made is really very great."

As Einstein wrote, "the clear conception of the differential law is one of Newton's greatest intellectual achievements". Newton's clear conceptions did not spring from nothing. The intuitions that elicited the demonstrations, applied over an extended period of time, constructing Newton's clear conception, are one of Newton's greatest strengths.

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